Exercise 7

Let z be a nonzero complex number and n a negative integer ($n = -1, -2, \ldots$). Also, write $z = re^{i\theta}$ and $m = -n = 1, 2, \ldots$. Using the expressions

$$z^m = r^m e^{im\theta} \quad \text{and} \quad z^{-1} = \left(\frac{1}{r}\right) e^{i(-\theta)},$$

verify that $(z^m)^{-1} = (z^{-1})^m$ and hence that the definition $z^n = (z^{-1})^m$ in Sec. 7 could have been written alternatively as $z^n = (z^m)^{-1}$.

Solution

Use the provided expressions to verify the result. Start with the left-hand side.

$$\begin{aligned}
(z^m)^{-1} &= (r^m e^{im\theta})^{-1} \\
&= \left(\frac{1}{r^m}\right) e^{i(-m\theta)}
\end{aligned}$$

Now simplify the right-hand side.

$$\begin{aligned}
(z^{-1})^m &= \left[\left(\frac{1}{r}\right) e^{i(-\theta)}\right]^m \\
&= \left(\frac{1}{r}\right)^m e^{i(-\theta)m} \\
&= \left(\frac{1}{r^m}\right) e^{i(-m\theta)}
\end{aligned}$$

Both sides yield the same result, so $(z^m)^{-1} = (z^{-1})^m$ is verified.